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GENERAL THEORY OF WINDMILLS.

By Max M. Munk.

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Summary.

In this paper, prepared for the National Advisory Committee for Aeronautics, the application of the slip curve method to the design and analysis of windmills is discussed and is illustrated by an example.

References.

1. Analysis of W. F. Durand's and E. P. Lesley's Propeller Tests. By Max M. Munk. N.A.C.A. Technical Report No. 175.

2. Analysis of Dr. Schaffran's Propeller Model Tests. By Max M. Munk. N.A.C.A. Technical Note No. 158.

3. The Analysis of Free Flight Propeller Tests and its Application to Design. By Max M. Munk. N.A.C.A. Technical Report No. 183.

4. Wind Driven Propellers. By Max M. Munk. N.A.C.A. Technical Memorandum No. 201.

Wind driven propellers are much used as small sources of power in an aircraft, e.g., for radio instruments and fuel pumps. In principle, they are nothing but ordinary windmills on a small scale. They differ considerably from ordinary windmills, however, because they are constructed from another viewpoint. In a windmill, the problem is to obtain the greatest possible horsepower at the least cost, it being a matter of indifference as to how much wind is utilized, for the wind costs nothing. In employing windmills on airplanes, however, all unnecessary retardation of the airplane must be avoided and the efficiency of such windmills can by no means be disregarded.

In Papers 1, 2, and 3, I presented a new method for the design or analysis of a propeller. Certain nominal slipstream velocities v and w are computed from the thrust or the torque, respectively, by the use of the following equation:

$$\text{Thrust coefficient} \quad C_T = \frac{\text{Thrust}}{D^2 \frac{\pi}{4} \quad V^2 \frac{\rho}{2}}$$

$$\text{Power coefficient} \quad C_P = \frac{\text{Horsepower}}{D^2 \frac{\pi}{4} \quad V^3 \frac{\rho}{2}}$$

where D denotes the propeller diameter, V the velocity of flight and ρ the density of air.

Then:

(1) the relative slip velocity computed from the thrust is

$$v/V = \sqrt{1 + C_T} - 1 \quad \text{or} \quad C_T = \left[1 + \frac{v}{V}\right]^2 - 1$$

(2) the relative slip velocity computed from the horsepower is connected with it by the equation

$$C_P = 2\,\frac{w}{V} + 2\left(\frac{w}{V}\right)^2 + \frac{1}{2}\left(\frac{w}{V}\right)^3$$

(3) the efficiency appeared to be

$$\frac{1}{1+\frac{w}{2V}} \; \frac{v/V}{w/V} \; \frac{1+\frac{v}{2V}}{1+\frac{w}{2V}}$$

The relative slip velocity when plotted against the relative tip velocity U/V, where U is the tangential velocity of the propeller tip, gave a straight line within the working range and this made the method valuable. All this is discussed in detail in the papers referred to.

This method, originally invented for propellers, can also be used to advantage in the design or the analysis of

Relative slip velocity computed from the thrust:

(4) $$\frac{v}{V} = 1 - \sqrt{1 - C_T}\;; \quad C_T = \frac{\text{Thrust}}{D^2 \, \frac{\pi}{4} \, V^2 \, \frac{\rho}{2}}$$

(5) $$C_T = 1 - \left(1 - \frac{v}{V}\right)^2$$

Relative slip velocity computed from the horsepower:

(6) $$C_P = \frac{\text{Power}}{D^2 \, \frac{\pi}{4} \, V^3 \, \frac{\rho}{2}} = 2\left(\frac{w}{V}\right) - 2\left(\frac{w}{V}\right)^2 + \frac{1}{2}\left(\frac{w}{V}\right)^3$$

Efficiency:

(7) $$\eta = \left(1 + \frac{w}{2V}\right) \frac{w/V}{v/V} \; \frac{1 - \frac{w}{2V}}{1 - \frac{v}{2V}}$$

This equation follows from $\eta = \frac{P}{TV}$ by substituting equations 1 and 2. The relation (6) is plotted in Fig. 2. From this curve w can be taken if C_P is known.

As an illustration of the application of these equations the values of v/V and w/V the relative slip velocities referring to the thrust and that referring to the horsepower, are computed from an actual test (Ref. 4) and in Fig. 1 they are plotted against the relative tip velocity U/V.

It appears that between $U/V = 2$ and 2.5 the slip curves so obtained are nearly straight and run parallel to and near each other as with propellers. The slip curve for the power is the lower one, the space between the two curves represents the losses in addition to the theoretical slip stream loss. Mean $U/V = 2$ the slip curve for the thrust has a break and runs then straight again but with a smaller slope. The points computed from the power are less regularly arranged. The mill did not rotate beyond $U/V = 2.5$ without being driven, in spite of the thrust having still the same direction.

In the papers referred to I gave the following approximate formula the slip modulus $m = \frac{d\ v/V}{d\ U/V}$, that is, the slope of the slip curve

(8).
$$m = \frac{2.8 \frac{S}{D^2}}{1 + 1.4 \frac{S}{D^2} \left(\frac{U}{V}\right)_0}$$

where S is the entire blade area and $(U/V)_0$ the intersecting point of the slip curve and the horizontal axis $v/V = 0$, $(U/V)_0 = 2.9$ in this case. With the propellers this formula gave results agreeing within a few percent of the observed value. The windmill investigated had six blades, the diameter D was .50 meters and the entire blade area was .055 sq.m. This gives $m = .36$. The curve computed from the test gives $m = .22$. That is less than two-thirds of the computed value, and a much larger disagreement between test and equation (4) than with the propellers.

I do not conclude from this, however, that the formula (4) is in itself less reliable for a windmill than it is for a propeller. The propeller test had a much smaller blade area ratio and only two blades. I suppose that the six blades, comparatively close together, interfered with each other, diminishing the produced lift and hence decreasing the value of the slip modulus. A propeller with six blades of the same shape would probably have a similar small slip modulus, and equation (4) would not hold better for it either.

Windmills used to have a larger relative blade area S/D^2, and therefore, more studies, on the magnitude of the value of the slip modulus m for such propeller shapes, are desirable. The value of m may be estimated, however, from the test quoted and from experiments with propellers with very wide blades. Then the procedure to design a windmill is quite similar to the design of a propeller. One point of the slip curve is computed from the desired R.P.M. and the velocity of flight and from the desired or estimated diameter. That slip curve can then be drawn which gives the least change of variation of revolutions, similarly as shown in Ref. 4. The intersection of the slip curve with the horizontal axis gives the pitch (Ref. 1 and 4) and the slope of the curve gives the area. This is discussed in the papers referred to, and can only be fully understood by actual trial. Only then will the simplicity of the method appear.

Fig. 1

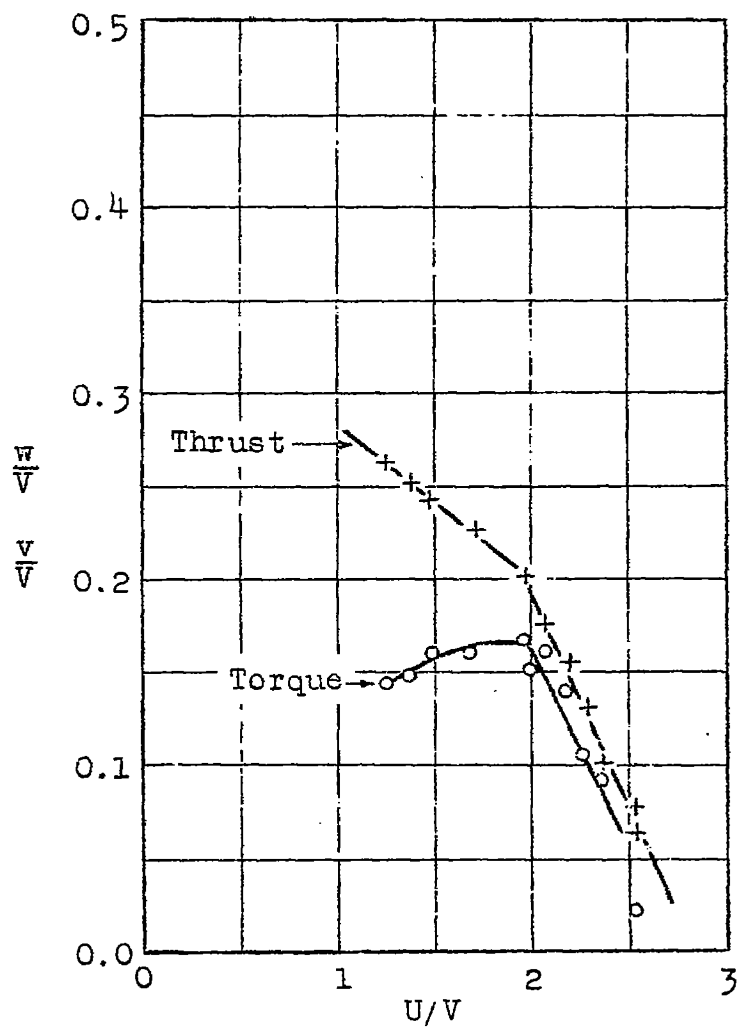

Fig. 1 Slip curves

Fig. 2

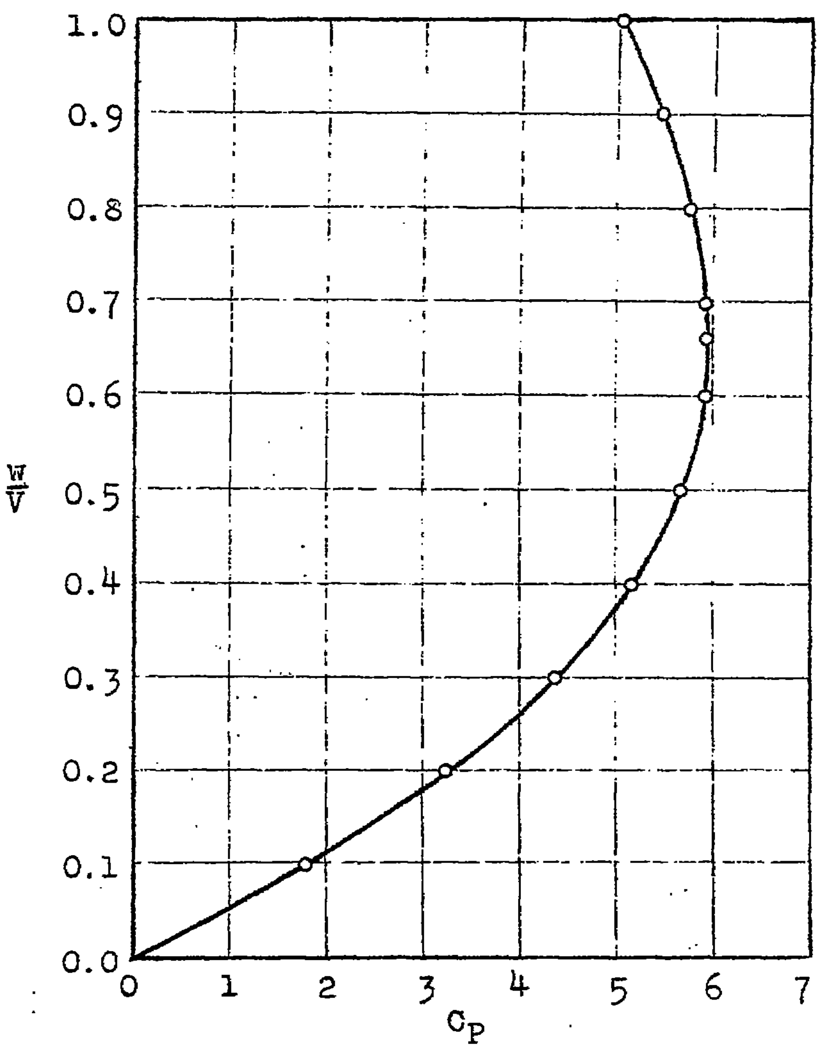

Fig. 2 $\frac{w}{V}$ against C_P